ISSN: 2277-9655
(I2OR), Publication Impact Factor: 3.785

IJESRT

INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH TECHNOLOGY

LS-SASAKIAN MANIFOLDS WITH SEMI-SYMMETRIC NON-METRIC CONNECTION

L K Pandey
D S Institute of Technology & Management, Ghaziabad (U.P.) 201007, India

ABSTRACT

In 1976, I. Sato [4] introduced a structure similar to almost contact structure. Also in 1977, I. Sato [5] discussed on a structure similar to almost contact structure II. In 1977, T. Adati and K. Matsumoto [1] discussed on conformally recurrent and conformally symmetric p-Sasakian manifolds. Also in 1979, K. Matsumoto and I. Sato [2] discussed on p-Sasakian manifolds with certain conditions. The purpose of this paper is to study Lorentzian special Sasakian manifolds and generalized Lorentzian Co-symplectic manifolds with semi-symmetric non-metric connection [3].

KEYWORDS: Nearly and almost LS-Sasakian manifolds, generalized L-Co-symplectic manifolds, semi-symmetric non-metric connection.

INTRODUCTION

An n-dimensional differentiable manifold M_n, on which there are defined a tensor field $F$of type (1, 1), a vector field T, a 1-form A and a Lorentzian metric g, satisfying for arbitrary vector fields $X, Y, Z, \ldots$

(1.1) $\overline{\overline{X}} = -X - A(X)T, \quad \overline{T} = 0, \quad A(T) = -1, \quad \overline{X} \stackrel{\text{def}}{=} FX, A(\overline{X}) = 0, \quad \text{rank } F = n - 1.$

(1.2) $g(\overline{X}, \overline{Y}) = g(X, Y) + A(X)A(Y)$, where $A(X) = g(X, T)$,

$`F(X, Y) \stackrel{\text{def}}{=} g(\overline{X}, Y) = -`F(Y, X),$

Then M_n is called a Lorentzian contact manifold (an L-Contact manifold).

Let D be a Riemannian connection on M_n, then we have

(1.3) (a) $(D_X`F)(\overline{Y}, Z) - (D_X`F)(Y, \overline{Z}) + A(Y)(D_XA)(Z) + A(Z)(D_XA)(Y) = 0$

(b) $(D_X`F)(\overline{Y}, \overline{\overline{Z}}) = (D_X`F)(\overline{\overline{Y}}, \overline{Z})$

(1.4) (a) $(D_X`F)(\overline{Y}, \overline{Z}) + (D_X`F)(Y, Z) + A(Y)(D_XA)(\overline{Z}) - A(Z)(D_XA)(\overline{Y}) = 0$

(b) $(D_X`F)(\overline{\overline{Y}}, \overline{\overline{Z}}) + (D_X`F)(\overline{Y}, \overline{Z}) = 0$

An L-Contact manifold is called a Lorentzian special Sasakian manifold (an LS-Sasakian manifold), if

(1.5) (a) $(D_XF)(Y) + A(Y)\overline{X} - `F(X, Y)T = 0 \Leftrightarrow (D_X`F)(Y, Z) + A(Y)`F(X, Z) - A(Z)`F(X, Y) = 0$

(b) $D_XT = \overline{\overline{X}}$

On this manifold, we have

(1.6) (a) $(D_X A)(\overline{Y}) = `F(X,Y) \Leftrightarrow$ (b) $(D_X A)(Y) = -g(\overline{X}, \overline{Y}) \Leftrightarrow$

Nijenhuis tensor in an L-Contact manifold is given by

(1.7) $`N(X,Y,Z) = \left(D_{\overline{X}} `F\right)(Y,Z) + \left(D_{\overline{Y}} `F\right)(Z,X) + (D_X `F)\left(Y,\overline{Z}\right) + (D_Y `F)(\overline{Z},X)$

Where

$$`N(X,Y,Z) \stackrel{\text{def}}{=} g(N(X,Y),Z)$$

NEARLY AND ALMOST LORENTZIAN SPECIAL SASAKIAN MANIFOLDS

An L-Contact manifold is called a nearly Lorentzian special Sasakian manifold (a nearly LS-Sasakian manifold), if

(2.1) $(D_X `F)(Y,Z) - A(Y)\ `F(Z,X) - A(Z)\ `F(X,Y)$

$= (D_Y `F)(Z,X) - A(Z)\ `F(X,Y) - A(X)\ `F(Y,Z)$

$= (D_Z `F)(X,Y) - A(X)\ `F(Y,Z) - A(Y)\ `F(Z,X)$

The equation of a nearly LS-Sasakian manifold can also be written as

(2.2) (a) $(D_X F)Y + (D_Y F)X + A(Y)\overline{X} + A(X)\overline{Y} = 0 \Leftrightarrow$

(b) $(D_X `F)(Y,Z) + (D_Y `F)(X,Z) - A(Y)\ `F(Z,X) + A(X)\ `F(Y,Z) = 0$

These equations can be modified as

(2.3) (a) $(D_X F)\overline{Y} + (D_{\overline{Y}} F)X + A(X)\overline{\overline{Y}} = 0 \Leftrightarrow$

(b) $(D_X `F)\left(\overline{Y},Z\right) - \left(D_{\overline{Y}} `F\right)(Z,X) - A(X)g\left(\overline{Y}, \overline{Z}\right) = 0$

(2.4) (a) $(D_X F)\overline{\overline{Y}} + (D_{\overline{\overline{Y}}} F)X - A(X)\overline{Y} = 0 \Leftrightarrow$

(b) $(D_X `F)\left(\overline{\overline{Y}},Z\right) - \left(D_{\overline{\overline{Y}}} `F\right)(Z,X) - A(X)\ `F(Y,Z) = 0$

(2.5) (a) $(D_X F)Y + (D_Y F)X - A(Y)\{\overline{D_X T} - (D_T F)X\} - A(X)\{\overline{D_Y T} - (D_T F)Y\} = 0 \Leftrightarrow$

(b)$(D_X `F)(Y,Z) + (D_Y `F)(X,Z) + A(Y)\left\{(D_X A)\left(\overline{Z}\right) - (D_T `F)(Z,X)\right\} + A(X)\left\{(D_Y A)\left(\overline{Z}\right) - (D_T `F)(Z,Y)\right\} = 0$

An L-Contact manifold is called an almost Lorentzian special Sasakian manifold (an almost LS-Sasakian manifold), if

(2.6) $(D_X `F)(Y,Z) + (D_Y `F)(Z,X) + (D_Z `F)(X,Y)$

$-2\{A(X)\ `F(Y,Z) + A(Y)\ `F(Z,X) + A(Z)\ `F(X,Y)\} = 0$

GENERALIZED LORENTZIAN CO-SYMPLECTIC MANIFOLDS

An L-Contact manifold is called a generalized Lorentzian Co-symplectic manifold (a generalized L-Co-symplectic manifold), if

(3.1) (a) $\overline{(D_X F)\overline{Y}} = 0 \Leftrightarrow$

(b) $(D_X `F)\left(\overline{Y}, \overline{Z}\right) = 0$

Inconsequence of (1.4) (a), this equation can also be written as

(3.2) (a) $(D_X F)Y - A(Y)\overline{D_X T} - (D_X A)(\overline{Y})T = 0 \Leftrightarrow$

(b) $(D_X `F)(Y, Z) + A(Y)(D_X A)(\overline{Z}) - A(Z)(D_X A)(\overline{Y}) = 0$

Therefore, a generalized L-Co-symplectic manifold is an LS-Sasakian manifold, if

(3.3) $D_X T = \overline{\overline{X}}$

An L-Contact manifold is called a generalized nearly Lorentzian Co-symplectic manifold (a generalized nearly L-Co-symplectic manifold), if

(3.4) $(D_X `F)(\overline{Y}, \overline{Z}) = (D_Y `F)(\overline{Z}, \overline{X}) = (D_Z `F)(\overline{X}, \overline{Y})$

Which implies

(3.5) $(D_X `F)(Y, Z) + A(Y)(D_X A)(\overline{Z}) - A(Z)(D_X A)(\overline{Y})$

$= (D_Y `F)(Z, X) + A(Z)(D_Y A)(\overline{X}) - A(X)(D_Y A)(\overline{Z})$

$= (D_Z `F)(X, Y) + A(X)(D_Z A)(\overline{Y}) - A(Y)(D_Z A)(\overline{X})$

Therefore, a nearly LS-Sasakian manifold is a generalized nearly L-Co-symplectic manifold, in which

(3.6) (a) $(D_X A)(\overline{Y}) = `F(\overline{X}, \overline{Y}) \Leftrightarrow$ (b) $(D_X A)(Y) = -g(\overline{X}, \overline{Y}) \Leftrightarrow$ (c) $D_X T = \overline{\overline{X}}$

An L-Contact manifold is called a generalized almost L-Co-symplectic manifold, if

(3.7) (a) $(D_X `F)(\overline{Y}, \overline{Z}) + (D_Y `F)(\overline{Z}, \overline{X}) + (D_Z `F)(\overline{X}, \overline{Y}) = 0 \Leftrightarrow$

(b) $(D_X `F)(\overline{\overline{Y}}, \overline{\overline{Z}}) + (D_Y `F)(\overline{\overline{Z}}, \overline{\overline{X}}) + (D_Z `F)(\overline{\overline{X}}, \overline{\overline{Y}}) = 0 \Leftrightarrow$

(c) $(D_X `F)(Y, Z) + (D_Y `F)(Z, X) + (D_Z `F)(X, Y) - A(X)\{(D_Y A)(\overline{Z}) - (D_Z A)(\overline{Y})\}$

$-A(Y)\{(D_Z A)(\overline{X}) - (D_X A)(\overline{Z})\} - A(Z)\{(D_X A)(\overline{Y}) - (D_Y A)(\overline{X})\} = 0$

Therefore, a generalized almost L-Co-symplectic manifold is an almost LS-Sasakian manifold, if

(3.8) $(D_X A)(\overline{Y}) - (D_Y A)(\overline{X}) = 2\, `F(X, Y)$

PROPERTIES

From (1.5), we see that for $\text{LS} - \text{Sasakian manifold}, D_T F = 0$. We will now consider nearly LS-Sasakian manifold

Putting T for X in (2.1), we get

(4.1) $(D_T `F)(Y, Z) = -(D_Y A)(\overline{Z}) + `F(Y, Z) = (D_Z A)(\overline{Y}) + `F(Y, Z)$

Hence

(4.2) (a) $(D_Y A)(\overline{Z}) + (D_Z A)(\overline{Y}) = 0 \Leftrightarrow$ (b) $D_T T = 0$

Barring Y and Z in equation (4.1) and using (1.4) (a) and (4.2), we get

(4.3) $(D_T`F)(Y,Z) = -\left(D_{\overline{Y}}A\right)(Z) - `F(Y,Z) = \left(D_{\overline{Z}}A\right)(Y) - `F(Y,Z)$

From (4.1) and (4.3), we obtain

(4.4) (a) $\left(D_{\overline{Y}}A\right)(Z) + (D_Z A)\left(\overline{Y}\right) = -2`F(Y,Z)$ (b) $(D_Y A)(Z) + (D_Z A)(Y) = -2g\left(\overline{Y}, \overline{Z}\right)$

Hence, on a nearly LS-Sasakian manifold, (4.1), (4.2), (4.3) and (4.4) hold.

Almost LS-Sasakian manifold will now be considered. Putting T for X in (2.6), we get

(4.5) (a) $(D_T`F)(Y,Z) = (D_Y A)\left(\overline{Z}\right) - (D_Z A)\left(\overline{Y}\right) - 2`F(Y,Z)$ $\Leftrightarrow$ (b) $D_T T = 0$

Barring Y and Z in equation (4.5) (a) and using (1.4) (a), we get

(4.6) $(D_T`F)(Y,Z) = \left(D_{\overline{Y}}A\right)(Z) - \left(D_{\overline{Z}}A\right)(Y) + 2`F(Y,Z)$

From (4.5) (a) and (4.6), we obtain

(4.7) (a) $\left(D_{\overline{Y}}A\right)(Z) - (D_Y A)\left(\overline{Z}\right) - \left(D_{\overline{Z}}A\right)(Y) + (D_Z A)\left(\overline{Y}\right) + 4`F(Y,Z) = 0 \Leftrightarrow$

(b) $\left(D_{\overline{Y}}A\right)\left(\overline{Z}\right) + \left(D_{\overline{Z}}A\right)\left(\overline{Y}\right) + (D_Y A)(Z) + (D_Z A)(Y) + 4g\left(\overline{Y}, \overline{Z}\right) = 0$

SEMI-SYMMETRIC NON-METRIC CONNECTION

Let us consider a connection B on M_n, defined by

(5.1) $B_X Y \stackrel{\text{def}}{=} D_X Y + A(Y)X$

The torsion tensor S of B is given by

(5.2) $S(X,Y) = A(Y)X - A(X)Y$

Further, if

(5.3) $(B_X g)(Y,Z) + A(Y)g(Z,X) + A(Z)g(X,Y) = 0,$

then B is called a semi-symmetric non-metric connection.

Put

(5.4) $B_X Y = D_X Y + H(X,Y)$

Where H is a tensor field of type (1, 2), then

(5.5) (a) $H(X,Y) = A(Y)X$

(b) $`H(X,Y,Z) = A(Y)g(X,Z)$

(c) $`S(X,Y,Z) = `H(X,Y,Z) - `H(Y,X,Z)$

Where

$$`H(X,Y,Z) \stackrel{\text{def}}{=} g(H(X,Y),Z)$$

$$`S(X,Y,Z) \stackrel{\text{def}}{=} g(S(X,Y),Z)$$

In an L-Contact manifold with the semi-symmetric non-metric connection B, it can be seen that

ISSN: 2277-9655
(I2OR), Publication Impact Factor: 3.785

(5.6) (a) $B_X T = D_X T - X$

(b) $(B_X A)(Y) = (D_X A)(Y) - A(X)A(Y)$

(c) $(B_X `F)(Y,Z) = (D_X `F)(Y,Z) + A(Y)`F(Z,X) + A(Z)`F(X,Y)$

(d) $(B_X `F)\left(\overline{Y},\ \overline{\overline{Z}}\right) = (B_X `F)\left(\overline{\overline{Y}}, \overline{Z}\right)$

(e) $(B_X `F)\left(\overline{\overline{Y}},\ \overline{\overline{Z}}\right) + (B_X `F)\left(\overline{Y},\ \overline{Z}\right) = 0$

(f) $`N(X,Y,Z) = (B_{\overline{X}} `F)(Y,Z) + (B_{\overline{Y}} `F)(Z,X) + (B_X `F)(Y,\overline{Z}) + (B_Y `F)(\overline{Z},X)$

Therefore,

An L-contact manifold is called a nearly LS-Sasakian manifold, if

(5.7) $(B_X `F)(Y,Z) - 2A(Y)`F(Z,X) - 2A(Z)`F(X,Y)$

$= (B_Y `F)(Z,X) - 2A(Z)`F(X,Y) - 2A(X)`F(Y,Z)$

$= (B_Z `F)(X,Y) - 2A(X)`F(Y,Z) - 2A(Y)`F(Z,X)$

And an L-contact manifold is called an almost LS-Sasakian manifold, if

(5.8) (a) $(B_X `F)(Y,Z) + (B_Y `F)(Z,X) + (B_Z `F)(X,Y) - 4\{A(X)\ `F(Y,Z) + A(Y)`F(Z,X) + A(Z)`F(X,Y)\} = 0$

This gives

(b) $(B_{\overline{X}} `F)(\overline{Y},\ \overline{Z}) + (B_{\overline{Y}} `F)(\overline{Z},\ \overline{X}) + (B_{\overline{Z}} `F)(\overline{X},\ \overline{Y}) = 0$

An L-Contact manifold is called a generalized L-Co-symplectic manifold, if

(5.9) $(B_X `F)(\overline{Y},\ \overline{Z}) = 0$

Inconsequence of (1.4) (a), (5.6) (b) and (5.6) (c), this equation can also be written as

(5.10) $(B_X `F)(Y,Z) + A(Y)(B_X A)(\overline{Z}) - A(Z)(B_X A)(\overline{Y}) - A(Y)`F(Z,X) - A(Z)`F(X,Y) = 0$

An L-Contact manifold is called a generalised nearly L-Cosymplectic manifold, if

(5.11) $(B_X `F)(\overline{Y},\ \overline{Z}) = (B_Y `F)(\overline{Z},\ \overline{X}) = (B_Z `F)(\overline{X},\ \overline{Y})$

Or

(5.12) $(B_X `F)(Y,Z) + A(Y)(B_X A)(\overline{Z}) - A(Z)(B_X A)(\overline{Y}) - A(Y)`F(Z,X) - A(Z)`F(X,Y)$

$= (B_Y `F)(Z,X) + A(Z)(B_Y A)(\overline{X}) - A(X)(B_Y A)(\overline{Z}) - A(Z)`F(X,Y) - A(X)`F(Y,Z)$

$= (B_Z `F)(X,Y) + A(X)(B_Z A)(\overline{Y}) - A(Y)(B_Z A)(\overline{X}) - A(X)`F(Y,Z) - A(Y)`F(Z,X)$

An L-Contact manifold is called a generalized almost L-Co-symplectic manifold, if

(5.13) (a) $(B_X `F)(\overline{Y},\ \overline{Z}) + (B_Y `F)(\overline{Z},\ \overline{X}) + (B_Z `F)(\overline{X},\ \overline{Y}) = 0$

(b) $(B_X `F)(\overline{\overline{Y}},\ \overline{\overline{Z}}) + (B_Y `F)(\overline{\overline{Z}},\ \overline{\overline{X}}) + (B_Z `F)(\overline{\overline{X}},\ \overline{\overline{Y}}) = 0$

(c) $(B_X `F)(Y,Z) + (B_Y `F)(Z,X) + (B_Z `F)(X,Y) - A(X)\{(B_Y A)(\overline{Z}) - (B_Z A)(\overline{Y})\}$

$-A(Y)\{(B_Z A)(\overline{X}) - (B_X A)(\overline{Z})\} - A(Z)\{(B_X A)(\overline{Y}) - (B_Y A)(\overline{X})\} -$

$2\{A(X)`F(Y,Z) + A(Y)`F(Z,X + A(Z)`F(X,Y))\} = 0$

from (3.3) and (5.6) (a), we see that a generalized L-Co-symplectic manifold is an LS-Sasakian manifold, if

(5.14) $B_X T = \overline{\overline{X}} - X$

Theorem 5.1 A nearly LS-Sasakian manifold is a generalized nearly L-Co-symplectic manifold, in which

(5.15) (a) $(B_X A)(\overline{Y}) = `F(\overline{X}, \overline{Y}) \Leftrightarrow$ (b) $(B_X A)(Y) + A(X)A(Y) = -g(\overline{X}, \overline{Y}) \Leftrightarrow$ (c) $B_X T = \overline{\overline{X}} - X$

Proof. Using (3.6) (a), (3.6) (b), (3.6) (c), (5.6) (a) and (5.6) (b) the result follows by simple computation.

Theorem 5.2 A generalized almost L-Co-symplectic manifold is an almost LS-Sasakian manifold, if

(5.16) $(B_X A)(\overline{Y}) - (B_Y A)(\overline{X}) = 2`F(X,Y)$

Proof. Making the use of (5.6) (b) and (3.8), we obtain (5.16)

COMPLETELY INTEGRABLE MANIFOLDS

Barring X, Y, Z in (5.6) (f) and using equations (5.7), (5.6) (d), we see that a nearly LS-Sasakian manifold is completely integrable, if

(6.1) $(B_{\overline{X}} `F)(\overline{Y}, \overline{\overline{Z}}) + (B_{\overline{Y}} `F)(\overline{Z}, \overline{\overline{X}}) = 0.$

Barring X, Y, Z in (5.6) (f) and using equations (5.8) (b), (5.6) (d), we can prove that an almost LS-Sasakian manifold is completely integrable, if

(6.2) $(B_{\overline{Z}} `F)(\overline{X}, \overline{\overline{Y}}) = 0.$

REFERENCES

[1] Adati, T. and Matsumoto, K., "On conformally recurrent and conformally symmetric p-Sasakian manifolds", TRU Mathematics, 13, pp. 25-32, 1977.
[2] Matsumoto, K. and Sato, I., "On p-Sasakian manifolds satisfying certain conditions", Tensor N.S., 33, pp. 173-178, 1979.
[3] Nirmala S. Agashe and Mangala R. Chafle, "A Semi-symmetric non-metric connection on a Riemmanian manifold", Indian J. pure appl. Math., 23(6), pp. 399-409, 1992.
[4] Sato, I., "On a structure similar to almost contact structure I", *Tensor N.S.,30*, pp. 219-224, 1976.
[5] Sato, I., "On a structure similar to almost contact structure II", *Tensor N.S.,31*, pp. 199-205, 1977.